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CALCULATING CONVECTIVE HEAT EXCHANGE IN A HYPERSONIC VISCOUS SHOCK LAYER

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The streamlining of bodies with catalytic surfaces is investigated within the framework of a model of a hypersonic three-dimensional viscous shock layer.

With the motion of a body on an entry glide path the segment of the trajectory subjected to heat stresses lies in the region of nonequilibrium dissociation in which consideration must be given to a variety of physicochemical processes. The solution of such problems within the scope of total Navier-Stokes equations involves considerable difficulties, even when using the latest computer equipment, and a solution has been found only for axisymmetric flows. In order to carry out mass calculations it is expedient to employ simplified models and approximate relationships which permit estimates of the solution with retention of acceptable accuracy.

The present paper covers an investigation into the flows of heat to an indestructible blunt-body surface, and this study is based on the equations of a three-dimensional hypersonic viscous shock layer [1]. The equations describing the flow contain terms from the equations for the boundary layer and the nonviscous shock layer in hypersonic approximation. A model for a hypersonic or thin viscous shock layer was first proposed in [2] for two-dimensional flows. This model is an asymptotic form of the Navier-Stokes equations for large Mach and Reynolds numbers, as well as for the density ratios behind and in front of the shock wave, which is characteristic of the main portion of the glide trajectory.

The nonequilibrium chemical reactions and the multicomponent diffusion are taken into consideration in these equations. Thermal and pressure diffusion can be neglected. It is assumed that the internal degrees of freedom are excited in equal measure.

An absence of heat flow to the body is assumed in the boundary conditions at the surface of an impermeable body, and the effects of catalytic atom recombination at the wall are taken into consideration. The generalized Rankine-Hugoniot relationships are used as the boundary conditions at the shock wave, and these relationships allow us to take into consideration the effects of molecular transfer within the shock-wave zone.

The method of numerical solution is analogous to that described in [1]. Unlike that particular reference, provision is made in the equations for all of the nonsequential space measurement terms, thus allowing us more exactly to determine the flows of heat in regions of lower Reynolds numbers than was the case in [1].

Let us examine the flow in the vicinity of the critical point of a convex blunt body which is the point at which a plane perpendicular to the velocity vector of the approaching flow is tangent to the surface of the body. The equation for the surface of the body in this vicinity can be approximated to an accuracy of second-order terms by the equation of an elliptic paraboloid:

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$$2y^3 = (y^1)^2 + k(y^2)^2, \quad k = R_1/R_2,$$

where R_1 and R_2 are the radii of the principal curves at the critical point, and for the direction of the y^3 axis we have chosen the direction of the velocity vector for the approaching flow. Let R_1 be the smaller of the curvature radii. The parameter k will then vary in a range from 0 to 1.

Figure 1 shows the influence exerted by the parameter k and the curvature radius on heat exchange. It follows from our numerical study that for nonequilibrium streamlining, as in the case of the flow of a homogeneous gas [4], the heat flow to the critical point of a double curvature can be determined approximately from the flow of heat to an axisymmetric critical point exhibiting curvature

$$\frac{1}{R} = \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right).$$

The catalytic activity of the surface and the parameters of the approaching flow must coincide in this case. It should be noted that the relationship between the relative heat flow $q(k)/q(1)$ and the parameter k varies for different points on the trajectory and in the models of the heterogeneous reactions [5].

Comparison of the limit case of two-dimensional streamlining ($k = 0$) and axisymmetric streamlining in the case of a curvature radius greater by a factor of two (curves 5 and 6) yields the greatest error. The difference in the heat flows does not exceed 5, 9, and 8%, while the equilibrium surface temperature for an ideal catalytic surface, a noncatalytic surface, and the model [6] of the heterogeneous reactions is 18, 28, and 25 K, respectively; the recombination coefficients in the model are dependent on temperature. The chemical composition of the air at the surface in this case is approximately identical, while the structure of the flow differs substantially. The shock wave in the case of two-dimensional flow is greater by a factor 2-2.5 than in the case of axisymmetric flow.

As the parameter k increases from 0 to 1, the accuracy in determining the flow of heat is improved. Thus, for $k = 0.4$, the heat flows, with an accuracy of 2%, and the equilibrium surface temperature, accurate to within 6 K, coincide with the quantities for the corresponding axisymmetric flow (calculations 3 and 4). However, if for purposes of the estimating calculations we employ the data from axisymmetric streamlining of a body with a radius equal to one of the radii of the principal curvatures of the three-dimensional body, this might result in significant error in the determination of the heat flow, particularly in that segment of the trajectory subjected to heat stresses. For example, when $k = 0.4$, $R_1 = 0.5$ m, the calculation of an axisymmetric body with a radius $R = 0.5$ m exaggerates the heat flow by more than 20 and 30%, respectively, for ideal-catalytic (curves 1 and 3) and noncatalytic surfaces. The equilibrium temperature in the heat-stress segment of the trajectory is higher than 90-105 K.

To improve accuracy in determining the flow of heat at a three-dimensional critical point on the basis of data from two-dimensional streamlining we can employ the following procedure. When $0.4 \leq k \leq 1$ we calculate the streamlining of an axisymmetric body of radius $R = 2R_1/(k + 1)$, and when $0 \leq k < 0.4$ we calculate the streamlining of a two-dimensional body with radius $R = R_1/(k + 1)$ (calculations 5 and 7). In this case, the heat flows will coincide with an accuracy of up to 2%.

Let us note that if for purposes of studying the streamlining of bodies moving at great altitudes we take into consideration the rate of slippage and the temperature jump at the surface (the form of the boundary conditions is given in [5]), then in this case for the determination of the heat flow to a double-curvature critical point we can employ the same degree of accuracy as in the method proposed above.

Maximum heat flows are attained in the streamlining of bodies with an ideal catalytic coating. As was mentioned in [1], the maximum heat flows can be estimated with an accuracy of up to 5%, if we make use of the results from calculations of the streamlining of bodies by a homogeneous two-atom gas (curves 1 and 2).

For a body with a given surface temperature the law of binary similarity $\rho_\infty R = \text{const}$ is valid in the region where dissociation is the predominant reaction [7]. The calculations carried out for $v < 10$ km/sec showed that the dimensionless heat flow as a function of $\rho_\infty R$ is represented by a single curve as R changes from 0.1 to 5 m all the way to $Re_\infty \sim 10^5$. When

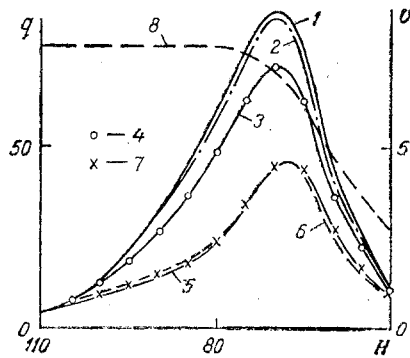


Fig. 1

Fig. 1. Heat flow (1-7) and velocity of body (8) as a function of flight altitude: 1, 2) $R_1 = 0.5$ m; $k = 1$; 3) 0.5 and 0.4; 4) 0.71 and 1; 5) 0.5 and 0; 6) 1 and 1; 7) 0.6 and 0.2; 8) data from [3]; 1, 3, 4) ideally catalytic surface; 2) homogeneous gas; 5-7) calculation for model [6] of heterogeneous reactions. q , 10^4 W/m²; v , km/sec; H , km.

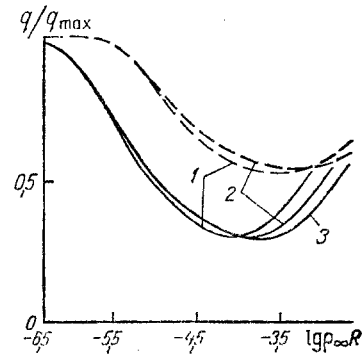


Fig. 2

Fig. 2. Ratio of heat-flow reduction in the case of a noncatalytic surface as a function of the Reynolds number, $k = 1$: 1) $R = 0.1$; 2) 1; 3) 5; solid lines) $v = 7.9$; dashed lines) 5. ρ_∞ , kg/m³.

we have the relationship between $\log \rho_\infty R$ and the dimensionless heat flow to the axisymmetric critical point, we can determine the flow of heat to the three-dimensional critical point with a smaller curvature radius R_1 by a simple shift of $\log(2/(k+1))$ along the axis of abscissas.

Of greatest interest in the study of the streamlining of an indestructible surface is the equilibrium temperature of the body. It is determined from the condition that the entire heat flow is radiated in equilibrium from a surface characterized by some emissivity. In this case, the relationship between the heat flow and $\rho_\infty R$ is separated for the various curvature radii immediately after cessation of the "frozen-in" flow. Thus, when $Re_\infty = 10^5$, $v = 5$ km/sec the dimensionless heat flow in the case of an ideal catalytic surface for $R = 5$ m is greater by 25% than when $R = 0.1$ m.

This study has demonstrated that for an equilibrium radiating surface the law of binary similarity can be used to determine the extent to which the flow of heat is reduced to a body with a constant rate of heterogeneous recombination. In this form the law of binary similarity is approximately satisfied (for the case shown in Fig. 2, with an accuracy of up to 6%) until the minimum value is attained for the heat flow as compared to the flow of heat to an ideal catalytic surface. This corresponds to the instant at which the degree of dissociation at the outer boundary of the viscous layer is maximum, while the recombination within the layer is almost completely frozen. With subsequent increase in the Reynolds number, the recombination process is intensified and the curves for the various curvature radii diverge. In using the model [6] for the progress of heterogeneous reactions, we find that the degree of reduction in the flow of heat depends significantly on the curvature radius.

We can use the results shown in Fig. 2 for the determination of the ratio of the reduction of the heat flow to the critical point of a double curvature, if we employ the method proposed above. Here the error in determining the heat flow to an ideal catalytic and noncatalytic surface is directed to one side while the ratio q/q_{\max} will be determined with an error that is smaller than the error in determining the dimensional heat flows.

NOTATION

y^1, y^2, y^3 , the Cartesian coordinate system; R_1, R_2, R , curvature radii at the critical point; k , ratio of the principal curvature radii; q , heat flow; v , velocity of flight; H , altitude; ρ_∞ , density of the incident flow; Re_∞ , Reynolds number calculated on the basis of the incident flow parameters.

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COMPARATIVE EVALUATION OF APPROXIMATE METHODS FOR SOLVING
ONE-DIMENSIONAL PROBLEMS INVOLVING MOVABLE BOUNDARIES

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Estimates of accuracy have been obtained for the most extensively employed approximation methods for the solution of the Stefan problem.

The overwhelming majority of problems involving unknown movable boundaries (problems of the Stefan-Verigin type) are not solved in quadratures. It thus becomes necessary to construct rather simple approximate solutions which might be used not only in estimation calculations, but also to verify the effectiveness of computational algorithms based on finite-difference methods. An attempt is made in this article, through comparison of existing analytical solutions, to evaluate the accuracy and scope of applicability for the most popular approximate methods of solution for Stefan problems in the case of a plane-parallel flow of heat: the Leibenzon-Charnyi method (LChM) [1, 2], the Barenblatt-Goodman integral method (IM) [3, 4], the successive approximation method (SAM) [5], and the combination method (CM) whose essence is explained below.

In the general case, the parabolic equation

$$\frac{\partial T}{\partial t} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial T}{\partial x} \right) \quad (1)$$

is satisfied by the following functions [7]:

$$\Phi(\zeta) = \operatorname{erfc}(\zeta), \operatorname{Ei}(-\zeta^2), \frac{\exp(-\zeta^2)}{\zeta} - \sqrt{\pi} \operatorname{erfc}(\zeta) \quad (2)$$

respectively, for $n = 0, 1, 2$. Here

$$\zeta = x/2 \sqrt{at}. \quad (3)$$

The solution of the specific boundary-value problem for Eq. (1) can be presented in the form

$$T = A + B\Phi(\zeta). \quad (4)$$

It is important here to underscore that A and B are not simply coefficients which must be determined from specific boundary-value problems, but rather integration constants. In par-

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